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FIXED POINTS UNDER ODD MAGIC SQUARE TRANSFORMATION

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Abstract-The existence of fixed points under a suitable transformation gives importance in mathematics and other sciences. We consider the existence of fixed points in magic squares. Magic squares are of odd order, singly even (divisible by 2) and doubly even (divisible by 4). The method for construction of Magic squares [6] based on Basic Latin Squares seems to be simpler. The input set (Initial Square) and output set (Magic square), developed for construction of odd magic square, using the algorithm [4] gives the only fixed point at its pivot element.
Keywords- Fixed point, magic square, MATLAB algorithm, pivot element, magic sum etc.

1. INTRODUCTION

A fixed point or an invariant point of a function $f(z)$ is an element of the function's domain that is mapped to itself by the function and can be determined by using the relation $f(z) = z$. In this case $f(z)$ is consider as the transformations for construction of magic square. The transformation $f(z) = \frac{az+b}{cz+d}$ where $ad - bc \neq 0$ has two fixed points if $(a-d)^2 + 4bc \neq 0$ and one fixed point if $(a-d)^2 + 4bc = 0$.

A magic square of order n is an arrangement of integers in an (n x n) matrix such that the sums of all the elements in every row, column and along the two main diagonals are equal. A normal magic square contains the integers from 1 to n^2 and exists for all orders $n \geq 1$ (except for n = 2). Magic squares are classified into 3 types: odd, singly even (divisible by 2), doubly even (divisible by 4). The magic sum of a (n x n) magic square is $S = \frac{n(n^2+1)}{2}$. Magic squares are used in puzzle games of cubes, pattern recognition and magic carpet constructions, magic square cipher in cryptology etc.

2. CONSTRUCTION OF ODD MAGIC SQUARE

There exist different methods or techniques for construction of odd magic squares based on algebra, graphs, computer oriented techniques etc. but Tomba (2012) [6] developed a simple technique for constructing odd-order magic squares using basic Latin squares and described as follows:

Step 1: Representing the input set on initial square (normal), finding the pivot element $P = \frac{n^2+1}{2}$

and magic sum $S = \frac{n(n^2+1)}{2}$

Step 2: Representing the initial square in Basic Latin Square format.

Step 3: Assignning the elements containing the pivot element as diagonal element and arranging in an orderly manner represents a Magic Square.

3. MODIFIED ALGORITHM [4] FOR CONSTRUCTION OF ODD MAGIC SQUARE

The above technique [6] is modified slightly to give the desired magic square by changing Step-3 only as:

Step-3 (modified): Fixed the column (or row) associated with the Pivot element. Perform double pass /steps of Basic Latin square construction with the remaining columns (or rows) and shift the fixed column (or row) in the middle gives the desired magic square.

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3.1 Program Coding (MATLAB)

```
% Initialisation of input matrix with 1 to n^2
 n=any odd number ≥3;
m=1;
for i=1:n
   for j=1:n
      a(i,j)=m;
      m=m+1;
   end
end

% Formation of basic Latin Square matrix

% shifting elements to left by i-1 position for each row
b=a;
for i=2:n
   for j=i:n
      l=i;
      for k=1:n
         b(i,k)=a(i,l);
         l=l+1;
         if l>n
            l=1;
         end
      end
   end
end

% shifting up of elements by j-1 position for each column
a=b;
for j=2:n
   for i=j:n
      l=j;
      for k=1:n
         b(k,j)=a(l,j);
         l=l+1;
         if l>n
            l=1;
         end
      end
   end

end

% Shifting to right by  (n+1)/2 position along row
a=b;
for i=1:n
     l=(n+1)/2;
      for j=1:n
         b(i,l)=a(i,j);
         l=l+1;
         if l>n
            l=1;
         end
      end
  end

% Displaying  b as the required magic square
b
```

4. CONCLUSION

Magic square generated from initial square with the program for n = 3, 5, 7, 9 and 13 are as follows:

1	2	3
4	5	6
7	8	9

3 x 3 input

8	1	6
3	5	7
4	9	2

3 x 3 output

Figure 1. Generation of 3 x 3 Magic square

1	2	3	4	5
6	7	8	9	10
11	12	13	14	15
16	17	18	19	20
21	22	23	24	25

5 x 5 input

17	24	1	8	15
23	5	7	14	16
4	6	13	20	22
10	12	19	21	3
11	18	25	2	9

5 x 5 output

Figure 2. Generation of 5 x 5 Magic square

1	2	3	4	5	6	7
8	9	10	11	12	13	14
15	16	17	18	19	20	21
22	23	24	25	26	27	28
29	30	31	32	33	34	35
36	37	38	39	40	41	42
43	44	45	46	47	48	49

7 x 7 input

30	39	48	1	10	19	28
38	47	7	9	18	27	29
46	6	8	17	26	35	37
5	14	16	25	34	36	45
13	15	24	33	42	44	4
21	23	32	41	43	3	12
22	31	40	49	2	11	20

7 x 7 output

Figure 3. Generation of 7 x 7 Magic square

1	2	3	4	5	6	7	8	9
10	11	12	13	14	15	16	17	18
19	20	21	22	23	24	25	26	27
28	29	30	31	32	33	34	35	36
37	38	39	40	41	42	43	44	45
46	47	48	49	50	51	52	53	54
55	56	57	58	59	60	61	62	63
64	65	66	67	68	69	70	71	72
73	74	75	76	77	78	79	80	81

9 x 9 input

47	58	69	80	1	12	23	34	45
57	68	79	9	11	22	33	44	46
67	78	8	10	21	32	43	54	56
77	7	18	20	31	42	53	55	66
6	17	19	30	41	52	63	65	76
16	27	29	40	51	62	64	75	5
26	28	39	50	61	72	74	4	15
36	38	49	60	71	73	3	14	25
37	48	59	70	81	2	13	24	35

9 x 9 output

Figure 4. Generation of 9 x 9 Magic square

1	2	3	4	5	6	7	8	8	10	11	12	13
14	15	16	17	18	19	20	21	22	23	24	25	26
27	28	29	30	31	32	33	34	35	36	37	38	39
40	41	42	43	44	45	46	47	48	49	50	51	52
53	54	55	56	57	58	59	60	61	62	63	64	65
66	67	68	69	70	71	72	73	74	75	76	77	78
79	80	81	82	83	84	85	86	87	88	89	90	91
92	93	94	95	96	97	98	99	100	101	102	103	104
105	106	107	108	109	110	111	112	113	114	115	116	117
118	119	120	121	122	123	124	125	126	127	128	129	130
131	132	133	134	135	136	137	138	139	140	141	142	143
144	145	146	147	148	149	150	151	152	153	154	155	156
157	158	159	160	161	162	163	164	165	166	167	168	169

13 x 13 input

93	108	123	138	153	168	1	16	31	46	61	76	91
107	122	137	152	167	13	15	30	45	60	75	90	92
121	136	151	166	12	14	29	44	59	74	89	104	106
135	150	165	11	26	28	43	58	73	88	103	105	120
149	164	10	25	27	42	57	72	87	102	117	119	134
163	9	24	39	41	56	71	86	101	116	118	133	148
8	23	38	40	55	70	85	100	115	130	132	147	162
22	37	52	54	69	84	99	114	129	131	146	161	7
36	51	53	68	83	98	113	128	143	145	160	6	21
50	65	67	82	97	112	127	142	144	159	5	20	35
64	66	81	96	111	126	141	156	158	4	19	34	49
78	80	95	110	125	140	155	157	3	18	33	48	63
79	94	109	124	139	154	169	2	17	32	47	62	77

13 x 13 output

Figure 5. Generation of 13 x 13 Magic square

For all odd magic squares, there exist only one fixed point i.e the pivot elemnt of the magic square. And the minimum element and maximum element of the magic square lies on the row or column associated with the pivot element.

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